Dutch Book Arguments

Bayesianism is a movement in epistemology and the philosophy of science that suggests that beliefs come in degrees. Dutch book arguments are a type of argument showing that rational agents must have degrees of belief that obey various principles. These arguments were introduced by Bruno de Finetti to show that for a rational agent, her degrees of belief in various propositions must obey the three axioms of probability theory. That is, every proposition has a non-negative probability, any proposition the agent is certain of has probability 1, and the probability of "*A* or *B*" is the sum of the probabilities of *A* and *B* if *A* and *B* are incompatible. Dutch books have also been used to establish further constraints on degrees of belief, and how agents should update them over time. The basic form of the argument is to find, for any agent who violates the proposed constraint, a set of bets that the agent finds individually favorable, but which collectively would guarantee that the agent loses money – this unfavorable collection of bets is the so-called "Dutch book". This entry outlines how Dutch book arguments work, shows how they can be used to support some of the basic tenets of Bayesianism, and discusses some of the problems they face.

The background of any Dutch book argument is the claim that (beyond the standard propositions attitudes of belief, knowledge, desire, and so on) there is a propositional attitude of "degree of belief", and an assumption connecting this degree of belief to the agent's evaluation of bets as favorable or unfavorable. The assumption is that for every

agent and every proposition, there is some number p that counts as her "fair price" for a bet on that proposition. What this means is that she is willing to pay any amount less than \$$p$ for a bet that wins her \$1 if A is true, while she will accept any amount greater than \$$p$ for a bet that *loses* her \$1 if A is true (or similarly \$(100p) for a bet that pays \$100). These two actions are respectively called "buying" and "selling" a bet on A. We typically think of gamblers buying bets and a casino selling them, but the Dutch book argument assumes each agent is willing to do either. This fair price p is then said to be the agent's "degree of belief" in A.

For example, let's say you have a degree of belief .6 that it will rain tomorrow in Seattle. The argument assumes that you will be willing to pay \$59.99 to place a bet where you win \$100 if it rains tomorrow in Seattle, but you will not be willing to pay \$60.01. However, you will be willing to accept if someone else pays you \$60.01 to place a bet where you have to give them \$100 if it rains tomorrow in Seattle. This is clearly an idealization – in some cases there will be a range of values at which you will be unwilling to either buy or sell, while in other cases there may be a range at which you are happy to do either. This is connected to the phenomenon of risk-aversion.

But if we do accept this idealization, then the Dutch book argument for the three probability axioms is straightforward. An agent's fair price for any bet should never be negative, because then she'd be willing to give away some money to sell a bet that could only make her give away more money. If the agent is certain that A is true, then her price

should be 1 – if it's lower then she's certain she would lose more than she'd be willing to sell the bet for, and if higher then she'd be willing to buy it for more than she could win. Finally, if *A* and *B* are incompatible, then her price for "*A* or *B*" should be the sum of her prices for *A* and *B*. To see this, consider an example. If your degree of belief that Speedy will win the horse race is .2, and your degree of belief that Dasher will win is .3, and your degree of belief that either Speedy or Dasher will win is .4, then someone can be sure to make money off you by making three bets. First, you pay them $19.99 for a bet that wins you $100 if Speedy wins. Then you pay $29.99 for a bet that wins you $100 if Dasher wins. Then they pay you $40.01 for a bet that costs you $100 if either Speedy or Dasher wins. At this point you have lost $9.97. But once the race is over, if either Speedy or Dasher wins, then you each pay each other $100, while no more money changes hands if a different horse wins. Thus, no matter what happens, you lose $9.97. If your price for "either Speedy or Dasher will win" had been .5 (the sum of your individual prices), then they couldn't do this.

One worry about this argument is that it only shows that an agent whose betting behavior violates the probability axioms has *financial* problems, and not that she has *epistemic* problems. But Brian Skyrms suggests that the Dutch book actually shows that the agent values *exactly the same situation* in two different ways, and is thereby incoherent. For the first two cases, this is quite clear, because she must view the situation both as a bet that she finds favorable, and also as a situation in which she is guaranteed to lose money. The third argument raises further issues because it involves multiple bets.

The Skyrmsian interpretation of the Dutch book argument also gives a way to avoid the problem of risk-aversion. Although there may not be any specific price that divides buying and selling as required for the argument, David Christensen suggests that an agent's degrees of belief must "sanction a price as fair", even though the agent's risk-aversion prevents her from buying or selling in a range around that price. Although an agent whose degrees of belief violate the probability axioms may (if she's risk-averse) never run into financial problems, she still implicitly evaluates a set of bets as fair even though they guarantee a loss.

One further difficulty with the interpretation of Dutch book arguments is understanding what it means to "guarantee" a loss. The argument above interprets this as meaning that the agent herself is certain that there would be a loss. Another popular interpretation is that the guarantee must be a *logical* guarantee of a loss – note that an agent can be certain of things that are not logically guaranteed, and may be uncertain of some logical necessities. Changing this notion of guarantee changes the conclusion of the argument – for instance, instead of requiring that an agent's degree of belief in something she is *certain* of be 1, the logical interpretation requires that an agent's degree of belief in any *logical truth* be 1. Different interpretations of "guarantee" will give rise to conclusions with different versions of the axioms for probability. The standard axioms (due to Kolmogorov) describe a probability function as defined on a collection of abstract sets. The interpretation suggested here interprets the elements of these sets as *epistemic*

possibilities for the agent on which the proposition is true, while the logical interpretation takes the elements to be *logical* possibilities on which the proposition is true.

Although there are several worries about Dutch book arguments, the basic conclusion that degrees of belief must obey the probability axioms (in some form or other) has been supported by various other arguments as well. However, there has been less independent support for the conclusions of other Dutch book arguments. (For those who are familiar with the notions, Dutch books have been used to support versions of the additivity axiom with infinitely many disjuncts rather than just two, as well as the Bayesian rule of learning by conditionalization, and many other supposed requirements of rationality for belief.)

The worries about Dutch books have helped clarify the nature of degrees of belief, and the constraints they ought to satisfy for rational agents. In the basic cases, these arguments are not the only arguments for the same conclusion, so the worries aren't too troublesome. But in some cases involving infinity, or the change of beliefs in response to evidence, much work remains to be done to show that these arguments really support the claimed conclusions.

Kenny Easwaran

See also: Allais Paradox; Decision Theory, Philosophical Perspectives; Rational Decision-Making, Computational Perspectives

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