Book Review

The central thesis of this book is a principle called Causal Finitism— roughly, for every event, there are only finitely many events in its causal history. This principle is defended as the best way to respond to a variety of paradoxes, and is said to have significant consequences for theology and the metaphysics of physics. The book as a whole, and the individual chapters, have a very user-friendly structure. Chapter 1 sets out the overall thesis of the book and sketches the outline of the arguments, while chapter 10 summarizes the arguments and evaluates which seem more or less compelling. Each chapter has a similar structure, opening with an introductory section outlining the arguments to come, and ending with a section giving an overall evaluation of the force of those arguments. Because infinity is involved, some chapters or sections involve difficult mathematics. These sections are marked with an asterisk (or two, in some cases!), and are always written in such a way that a reader can skip them without losing too much of the thread.

Chapters 2–6 present various paradoxes of infinity, building the central (partially abductive) argument of the book. In each chapter, some family of paradoxes is considered. In each case, it is shown that Causal Finitism would eliminate the possibility of the paradox. Pruss considers other principles that would resolve the relevant paradoxes, but argues that these principles either leave some instances unresolved or would bar other things that are intuitively possible, so that Causal Finitism is the best resolution. Chapter 7 gives a more detailed account of just how Causal Finitism should be specified and includes an extended comparison with a competing principle from Michael Huemer's recent book, *Approaching Infinity* (New York: Palgrave Macmillan, 2016). Chapters 8 and 9 then explore the implications of Causal Finitism for contemporary physics and for theistic metaphysics, aiming to show that some apparent objections to it fail.

Despite the generally helpful structure, I could still have used some more signposts for the overall structure of the book. It was clear throughout that chapter 7 was going to include some of the most specific clarifications of the main thesis, but I didn't understand until I got to them how different chapters 8 and 9 would be from the others. My guess is that many potential readers will find better ways to engage with this book than just to open it

and read it from cover to cover as I did—one can easily read just chapters 1 and 10 to get the big picture and then jump to any chapter and read its first and last sections to get a sense of its argument, and only then dive into the middle sections to get more of the details. Fortunately, the table of contents includes page numbers for every chapter, section, subsection and subsubsection, so that a reader can quickly find whichever discussion they are looking for.

The paradox that opens the book is that of Thomson's lamp. A lamp is connected to a toggle button that turns it on if it is off and off if it is on. The lamp starts out off at 10 a.m., and the button is pushed once at 10:30, again at 10:45, and again at each moment that is $1/2^n$ hours before 11 a.m. The classic question is what the state of the lamp will be at 11 a.m., after the button has been pushed infinitely many times. As Pruss notes, there are various ways to avoid any suggestion of paradox here—one can note that actual physical laws prevent the button from being pushed fast enough for some of the later toggles; or one can just allow that the situation as described is indeterministic, so that nothing determines which state the lamp ends up in; or one can claim that there is some brute metaphysical law that prevents such an infinite configuration of pushes from occurring.

Because of the multiplicity of possible responses, Pruss doesn't rest much weight on this paradox, but uses it to help clarify the view. He notes that if the pushes of the button occurred once per second, forever, there would be no paradox—it's only when infinitely many pushes occur in a finite amount of time that there is a potential problem. Thus he proposes that no event can have infinitely many other events in its causal history. He allows that the future can be infinitely long, but the past must be finite (ch. 3, §5). Although space can be infinitely large, there can't be infinitely many events that collaborate to produce an event at a single later point in time. Perhaps more surprisingly, although time is infinitely divisible, so that there are infinitely many *potential* moments in any time interval, there are *actually* only finitely many moments in any such interval. (This idea is confusingly gestured at in §§4 and 5 of chapter 2, but worked out in fuller detail in chapter 8.)

There are other very clever paradoxes Pruss puts more weight on. Chapter 3 contains several related to Thomson's lamp. The one he puts most weight on also involves a similar lamp. The lamp is not on at 10 a.m., but there are a bunch of lamplighters, each of whom checks the lamp at some time during the hour and turns it on if it is not yet on, but otherwise does nothing, leaving it on. Thus the lamp will be on by 11 a.m. But if there is one lamplighter for each natural number, and the nth lamplighter checks at $1/2^n$ hours after 10 a.m., then for each lamplighter, by the time they check, the lamp will already be on (since other lamplighters checked earlier), and so the lamp will have to be on without any particular lamplighter being the one to turn it on. (For historical reasons, Pruss calls this the 'Grim Reaper' paradox, because other authors have discussed an example involving killing a person,

rather than turning on a lamp. There are also related paradoxes involving gods or boards preventing a person from walking in a particular direction.) The one paradox in this chapter I hadn't encountered in any form before (though it goes back at least to Raymond Smullyan) involves an infinitely strong rod that stretches infinitely far in one direction on an impenetrable surface. If the end of the rod is lifted a finite distance, then it will be impossible for the rod to tilt (since any non-horizontal angle would involve the solid rod penetrating the surface), so that the rod levitates with support only at one end.

Chapters 4 and 5 present a variety of paradoxes and problems involving infinity in probability and decision theory, which get more mathematically complicated. Chapter 4 discusses several paradoxes arising from the case of an infinite lottery picking a natural number, where each natural number has an equal chance of being picked. In any such lottery, the probability of a given number being picked must be less than any positive real value. Furthermore, for any particular natural number n, no matter how large, the probability of picking a number less than n must *also* be less than any positive real value. As a result, the probability that some number or other is picked (which is 1) is not equal to the limit of the probability that a number less than n is picked, so this probability distribution violates 'continuity' or 'countable additivity'. Any such probability distribution has a number of paradoxical features. If there are two such lotteries, then you can know in advance that learning the value produced by either lottery will strongly confirm the hypothesis that the other lottery produced a higher number. There are violations of 'conglomerability' and other related principles for probability. I have argued in several papers that such distributions should never arise for epistemic probability, but in this chapter Pruss shows that corresponding metaphysical problems follow from violations of Causal Finitism.

Chapter 5 continues with various infinite decision problems. If a die is rolled finitely many times, and you get to observe previous rolls and then attempt to guess the next, it is possible that you always get the wrong answer. However, if the die has been rolled *infinitely* many times, and you get to observe the entire infinite string of previous results before guessing each one, then (using the Axiom of Choice) there is a strategy that guarantees that you get the wrong answer only finitely many times! Furthermore, if you are offered infinitely many yes-no choices, where you get a benefit of size $1/2^n$ for each time you choose 'yes', but a penalty of 2 for choosing 'yes' infinitely many times, then there is no sequence of choices you make that can be rational. Again, Pruss shows how these difficulties are possible if there are violations of Causal Finitism, and impossible if it is satisfied. Chapter 6 in particular describes how a machine could be built to perform calculations that depend essentially on the Axiom of Choice, assuming the falsity of Causal Finitism.

I do not work primarily in metaphysics, so I am not the target audience for this book. I found much of the argumentation in chapters 2–7 unconvincing,

because it kept depending on the reader sharing certain intuitions about metaphysical possibility or impossibility, motivated by recombination principles that seemed somewhat obscure to me. For instance, Pruss argues against one alternative principle by saying, 'to argue for a thesis as controversial as the impossibility of immaterial minds on the grounds of paradoxes of infinity seems to be an overreach, especially given the alternative of causal finitism' (p. 159). But as early as page 10, and at many other points throughout the book, Pruss claims it is a virtue of Causal Finitism that it entails a finite past but allows for an infinite future. It's not at all clear to me why we should find the possibility of immaterial minds or an infinite future to be so much more compelling than the possibility of an infinite past or causal loops. But I can see that there might be a world-view on which this is natural, and this book is dedicated to exploring that world-view.

The part of the book I found most fascinating was chapter 8, which discusses how Causal Finitism interacts with physics (both classical and contemporary). One of the biggest challenges to Causal Finitism is that it entails that there are only finitely many events that are causal antecedents of any event, which in particular seems to entail that time is discrete, which further suggests that space must be discrete as well. However, a natural view of discrete time and space, where there is an evenly spaced lattice of points, picks out a preferred set of directional axes, and seems to imply that any object changes size and shape whenever it rotates. (Consider what happens when an object covering a 4×4 square area of a chessboard is rotated by 45 degrees.)

While none of this is clearly impossible, Pruss's development of an alternative involving an *irregular* discrete set of times and places is much more interesting. On the view he develops here, the points of space and time that exist are the ones at which something changes. If an object moves or rotates, the set of points that exists at the later time won't correspond exactly to the earlier one, but will follow the natural shape of the object. Any interval *can* be subdivided, but it *is* subdivided only finitely many times. He shows how this view is compatible with fundamental physics that works with differential equations, and also demonstrates its affinity with some contemporary interpretations of quantum mechanics (particularly Bell's 'flash' ontology). Whatever one thinks of Causal Finitism and the arguments for it, this development of a view of discrete spacetime that is compatible with all sorts of physical theory is worth learning about!

Chapter 9 explores the implications of Causal Finitism for theism. On the one hand, Causal Finitism yields very quickly the conclusion that every event must be (partially) caused by an uncaused cause, and if every contingent being is caused, then this means that these uncaused causes must be necessary beings. So Causal Finitism is exactly the sort of premiss that much traditional theological argumentation starts from. On the other hand, if there are infinitely many goods that form part of the reason why God created this world, or

if there are infinitely many free choices of agents about which God is om-
niscient, then Causal Finitism sits poorly with a causal view of reasons or
knowledge. Pruss argues that there are ways around these apparent difficul-
ties. But even more than the rest of the book, this chapter is exploratory, and
suggests further research programmes for working out the details.

Overall, I don't find myself convinced of the central thesis of the book.
However, it does seem quite interesting to me to see how views about theism,
physics, probability, and various paradoxes of infinity are tied together.
Metaphysicians with interests in these topics will want to look at least at
the relevant parts of this book. It's unfortunate that only the six pages of
§3.4 of chapter 7 are devoted to a comparison with Michael Huemer's view,
which blocks the paradoxes by banning infinite intensive magnitudes in phys-
ics. But a fuller comparison of these two views will be a natural next step for
people interested in these issues.

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