Philosophy 240, Kenny Easwaran
Sample Final 1

December, 2017

Name: Sample Student Section: ________________

1. Validity. Make up an argument with the described premises and conclusion, or say why such an argument is impossible. (5 pts each)

   (a) Invalid, with two true premises, and a false conclusion.

   Texas is a state. Every state has a capital. Therefore, Dallas is the capital of Texas.

   (b) Valid, with one true premise (the only premise), and a false conclusion.

   This is impossible, because a valid argument cannot have true premises and a false conclusion.

   (c) Valid, with one true premise, one false premise, and a true conclusion.

   All mammals are warm-blooded. Ducks are mammals. Therefore ducks are warm-blooded.

2. Translations

   (a) Translate the following sentences from English into the formal language of Tarski's World. (5 points each)

   i. b is the same size as a if a is a cube.

   Cube(a) -> SameSize(b,a)

   ii. b is not a large tetrahedron.

   ~(Large(b)^Tet(b))

   iii. Either c is larger than a, or d is.

   Larger(c,a) v Larger(d,a)

(b) Give ordinary English translations of the following sentences in the formal language of Tarski's World. (5 points each)

i. Dodec(b)$\rightarrow$ $\neg$Dodec(a)

If b is a dodecahedron then a is not. a is not a dodecahedron if b is.

b is a dodecahedron only if a isn't.

ii. Larger(a, b)$\vee$Larger(c, b)

Either a or c is larger than b. Either a is larger than b or c is.

iii. (Cube(b)$\wedge$Small(b))$\wedge$(Tet(c)$\wedge$Small(c))

b is a small cube but c is a a small tetrahedron. b is a small cube and c is a small tet.

(c) Translate the following sentences from English into the formal language of Tarski's World. (5 points each)

i. There is a small thing that is not to the left of a.

Ex(Small(x)^~LeftOf(x,a))

ii. If there is a large cube, then there is a small tetrahedron.

Ey(Large(y)^Cube(y)) -> Ey(Small(y)^Tet(y))

iii. Every tetrahedron adjoins a cube.

Ax(Tet(x) -> Ey(Cube(y) ^ Adjoins(x,y))

(d) Give ordinary English translations of the following sentences in the formal language of Tarski's World. (5 points each)

i. $\forall z$(Tet(z)$\rightarrow$ $\neg$Small(z))

No tetrahedron is small. Every tetrahedron is not small.

ii. $\forall z$(Cube(z)$\rightarrow$LeftOf(z, a))$\wedge\exists y$(Tet(y)$\wedge$RightOf(y, a))

Every cube is left of a, but there is a tetrahedron right of a.

All cubes are to the left of a, and there is a tetrahedron to the right of a.

iii. $\exists y$(SameRow(y, a)$\wedge\exists x$(Tet(x)$\wedge$SameSize(x, y)))

Something in the same row as a is the same size as a tetrahedron.

There is something in the same row as a that is the same size as some tetrahedron.

Name: ______________________________ Section: ______________________________

3. Assessing arguments

   (a) Construct a truth table for this argument and use it to state whether the argument is tautologically valid. (15 points)
   $B \to \neg C$. $A \vee C$. Therefore, $A \vee B$.

A	B	C	B->~C		A v C	A v B
T	T	T	F	F	T	T
T	T	F	T	T	T	T
T	F	T	T	F	T	T
T	F	F	T	T	T	T
F	T	T	F	F	T	T
F	T	F	T	T	F	T
F	F	T	T	F	T	F
F	F	F	T	T	F	F

This argument is not valid, because the seventh row makes both premises true and the conclusion false.

   (b) Fill in the rules and line numbers for this formal proof. (15 points)

1	$C \vee \neg B$	______
2	$C \to A$	______
3	$\quad B$	______
4	$\quad\quad C$	______
5	$\quad\quad A$	-> Elim 2,4
6	$\quad\quad \neg B$	______
7	$\quad\quad \bot$	_l_ Intro 3,6
8	$\quad\quad A$	_l_ Elim 7
9	$\quad A$	v Elim 1, 4-5, 6-8
10	$B \to A$	-> Intro 3-9

(c) Say whether or not this argument is valid. If it is valid, explain why, and if it is not valid, show this by describing a counterexample. (15 points)

$\exists y(\text{Cube}(y) \wedge \neg \text{Large}(y))$. $\text{Large}(a)$. Therefore, $\neg \text{Cube}(a)$.

This argument is not valid. To see this, consider a world where a is a large cube and b is a small cube. The first premise says that there is a cube that is not large, which is true because of b. The second premise says that a is large, which is true. The conclusion says that a is not a cube, which is false. Since this world makes the premises both true and the conclusion false, the argument is not valid.

(d) Say whether or not this argument is valid. If it is valid, explain why, and if it is not valid, show this by describing a counterexample. (15 points)

$\forall z(\text{Dodec}(z) \rightarrow \text{Large}(z))$. $\text{Tet}(a)$. Therefore, $\exists x \neg \text{Large}(x)$.

This argument is not valid. Consider a world that only consists of two objects, a and b, where a is a large tetrahedron and b is a large dodecahedron. The first premise says that every dodecahedron is large, which is true because b is large. The second premises says that a is a tetrahedron, which is true. But since every object is large, the conclusion is false, which says that something isn't large. Since this world makes the premises both true and the conclusion false, the argument is not valid.

This argument is not valid. Consider a world in which a is the only object, and it is a large tetrahedron. The first premise says that every dodecahedron is large, which is vacuously true because there is no dodecahedron at all. The second premises is true because a is a tetrahedron. Since a is the only object, and it is large, the conclusion is false. Thus, the argument isn't valid.

Name: ______________________________ Section:______________________________

(e) Devon picked up a coin at the magic shop. He wants to know if it is a fair coin, or one of the special biased coins at the shop, so he flips it. 80% of the coins are fair and 20% are biased. He knows that a fair coin would have a 50% chance of coming up heads, and a biased coin would have an 80% chance of coming up heads. What is the probability that the coin is fair given that the coin came up tails? How strong is this evidence as an argument that the coin is fair? (15 points)

Coin is fair.	Coin comes up heads.	Probability
T	T	.4
T	F	.4
F	T	.16
F	F	.04

80% of coins are fair, and of those, half come up heads and half come up tails, so the first two rows of the chart are .4 and .4. Out of the coins that are not fair (which is 20%), 80% come up heads, so the third row is
.2 x .8 = .16, and the fourth row is what is left, or .04.

P(Fair given Tails) = P(Fair and Tails)/P(Tails) = .4/.44 = 10/11, which is about 90.9%.

The original probability of being fair was 80%, so this piece of evidence that increased the probability above 90% is not very strong.