Philosophy 240, Kenny Easwaran
Sample Final 2

December, 2017

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1. Validity. Make up an argument with the described premises and conclusion, or say why such an argument is impossible. (5 pts each)

   (a) Valid, with two false premises, and a true conclusion.

   Every television show has a female lead character. Tchaikovsky's opera "Joan of Arc" is a television show. Therefore Tchaikovsky's opera "Joan of Arc" has a female lead character.

   (b) Valid, with two true premises, and a false conclusion.

   This is impossible because a valid argument is one that can't have true premises and a false conclusion.

   (c) Invalid, with one false premise, one true premise, and a false conclusion.

   All cats are mammals. Reveille is a mammal. Therefore Reveille is a cat.

2. Translations

   (a) Translate the following sentences from English into the formal language of Tarski's World. (5 points each)

      i. b is not a large cube if c is.

      (Large(c)^Cube(c)) -> ~(Large(b)^Cube(b))

      ii. a is either larger than b or the same size as it.

      Larger(a,b) v SameSize(a,b)

      iii. c and d aren't both small.

      ~(Small(c)^Small(d))

(b) Give ordinary English translations of the following sentences in the formal language of Tarski's World. (5 points each)

i. $\text{Tet}(a) \wedge \neg \text{Small}(a)$

a is a tetrahedron but it isn't small.
a is a tetrahedron that isn't small.

ii. $(\text{Large}(b) \wedge \text{Cube}(b)) \rightarrow (\text{Medium}(c) \wedge \text{Cube}(c))$

If b is a small cube then c is a medium one.
c is a medium cube if b is a small one.
b is a large cube only if c is a medium cube.

iii. $(\text{Cube}(d) \wedge \text{Small}(d)) \vee (\text{Cube}(a) \vee \text{Small}(a))$

Either d or a is a small cube.
Either d is a small cube or else a is.

(c) Translate the following sentences from English into the formal language of Tarski's World. (5 points each)

i. Not everything larger than c is a cube.

~Az(Larger(z,c) -> Cube(z))

ii. Some cube is larger than b if c is larger than every tetrahedron.

Ay(Tet(y) -> Larger(c,y)) -> Ex(Cube(x) ^ Larger(x,b))

iii. There is something to the left of d that is larger than something to the right of d.

Ex(LeftOf(x,d) ^ Ey(RightOf(y,d) ^ Larger(x,y))

(d) Give ordinary English translations of the following sentences in the formal language of Tarski's World. (5 points each)

i. $\exists y(\text{Small}(y) \wedge \neg \text{Tet}(y))$

Some small thing is not a tetrahedron.
There is something that is small but not a tetrahedron.

ii. $\forall z(\text{SameSize}(z, b) \rightarrow \text{Cube}(z)) \vee \forall z(\text{Larger}(z, b) \rightarrow \text{Cube}(z))$

Either everything the same size as b is a cube or everything larger than b is a cube.
All objects the same size as b are cubes, or else all objects larger than b are.

iii. $\forall x(\text{Larger}(x, a) \rightarrow \exists z(\text{Cube}(z) \wedge \text{SameRow}(z, x)))$

Everything larger than a is in the same row as a cube.
For all things larger than a, there is a cube in the same row as it.

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3. Assessing arguments

(a) Construct a truth table for this argument and use it to state whether the argument is tautologically valid. (15 points)
$C \to \neg B$. $A \vee C$. Therefore, $B \vee \neg A$.

A	B	C	C -> ~B	A v C	B v ~A
T	T	T	F F	T	T F
T	T	F	T F	T	T F
T	F	T	T T	T	F F
T	F	F	T T	T	F F
F	T	T	F F	T	T T
F	T	F	T F	F	T T
F	F	T	T T	T	T T
F	F	F	T T	F	T T

This argument is not valid, because the third row makes both premises true and the conclusion false. (The fourth row does too!)

(b) Fill in the rules and line numbers for this formal proof. (15 points)

1	$\neg B \vee (A \to C)$	_____
2	$B \wedge A$	_____
3	$\neg B$	_____
4	B	^ Elim 2
5	$\bot$	_|_ Intro 3,4
6	C	_|_ Elim 5
7	$A \to C$	_____
8	A	^ Elim 2
9	C	-> Elim 7,8
10	C	v Elim 1, 3-6, 7-9
11	$(B \wedge A) \to C$	-> Intro 2-10

(c) Say whether or not this argument is valid. If it is valid, explain why, and if it is not valid, show this by describing a counterexample. (15 points)
$\forall z(\text{Dodec}(z) \rightarrow \neg\text{Small}(z))$. $\text{Small}(b)$. Therefore, $\neg\text{Dodec}(b)$.

This argument is valid. To see this, do a proof by contradiction. If we assumed that b was a dodecahedron, then the first premise, which says every dodecahedron is not small, would mean that b is not small. But this would contradict the second premise. Since assuming b is a dodecahedron leads to a contradiction, we thus know that b must not be a dodecahedron, which is what the conclusion says.

(d) Say whether or not this argument is valid. If it is valid, explain why, and if it is not valid, show this by describing a counterexample. (15 points)
$\exists x\text{Small}(x)$. $\exists x\text{Tet}(x)$. Therefore, $\exists x(\text{Small}(x) \wedge \text{Tet}(x))$.

This argument is not valid. To see this, consider a world where a is a small cube and b is a large tetrahedron, and there are no other objects. In this world, the first premise is true because a is small. The second premise is true because b is a tetrahedron. The conclusion is false, because there is no small tetrahedron. Since the premises are true and the conclusion false in this world, the argument is not valid.

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(e) Mary is going to the doctor for her annual cancer screening. She knows that 1% of all women her age develop cancer in a given year. The doctor has given her a test that has a 90% chance of coming back positive if she does have cancer, and a 5% chance of coming back positive if she doesn't. What is the probability that she has cancer, given that her test comes back positive? How strong is this evidence as an argument that she has cancer? (15 points)

Cancer.	Positive test.	Probability
T	T	.9%
T	F	.1%
F	T	4.95%
F	F	94.05%

Out of the 1% of women who develop cancer in a given year, 90% (or a total of 0.9% of all women) get a positive test result, so the first row has 0.9%. The second row must have the remainder of that 1%, or 0.1%. Out of the 99% of women that don't develop cancer in the year, 5% (for a total of 0.99 x 5% = 4.95%) have a positive test result, and the remainder (94.05%) don't.

P(Cancer given Positive test) = P(Cancer and Positive test)/P(Positive test) = 0.9%/(4.95% + .9%) = .9/5.85 = about .1538.

The prior probability of cancer was 1%, and the positive test result brings it up to about 15%. This is not an especially strong argument, but because cancer is such a bad possibility, we might find it worrying and worth paying attention to anyway.